



GUESS PAPER

B.Tech I Semester

Engineering Mathematics – I/1FY2-01

SHORT QUESTIONS INTERGRAL CALCULAS UNIT-01

Q.1	Define β and γ function.
Q.2	Find the value of $\int_0^{\pi/2} \sin^6 \theta \cos^7 \theta d\theta$
Q.3	Find the value of $\Gamma(-1)$ & $\Gamma(-3)$ & $\Gamma(1)$ $\Gamma(3)$.
Q.4	Prove $\int_0^{\infty} e^{-ax} x^{n-1} dx = \frac{\Gamma(n)}{a^n}$.
Q.5	Evaluate $\int_0^{\infty} x^6 e^{-2x} dx$ using β and γ function.
Q.6	Write the formula of volume & surface area of solid of revolution when revolution is about x-axis.

SHORT QUESTIONS SEQUENCE AND SERIES

UNIT-02

Q.1	Write Comparison test , D Alembert's Ratio Test & Rabee's Test .
Q.2	Define Convergence of Power series.
Q.3	In the Taylor series expansion of e^x about $x=2$, the coefficient of $(x-2)^4$ is....
Q.4	Find whether series $\sum_{n=10}^{\infty} n$ is convergent or not.
Q.5	Determine the radii of convergence of power series $\sum_{n=1}^{\infty} \frac{2n!}{(n!)^2} z^n$
Q.6	Write the condition for p series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ to convergent and divergent.
Q.7	Define Convergence and Divergence of Geometric series $1+x+x^2+x^3+\dots$



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Q.8

Prove that the sequence $\langle x_n \rangle$;

Where $x_n = \frac{2n-7}{3n+2}$ is

- (i) Monotonically increasing
- (ii) Bounded
- (iii) It's limit is $\frac{2}{3}$



Q.9	Give one example of each (i) Monotonically increasing & convergent sequence (ii) Monotonically increasing & divergent sequence. (iii) Monotonically decreasing & Convergent sequence. (iv) Monotonically decreasing & divergent sequence.
Q.10	Find the limit of the sequence $\langle x_n \rangle$: where $x_n = \frac{5n-3}{7n+8}$
Q.11	Write the power series expansion of $\sin x$, $\cos x$, $\log(1+x)$, e^x .
Q.12	What do you mean by convergence of a sequence?
Q.13	Check the convergence $\frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \dots$

SHORT
QUESTIONS
FOURIER SERIES

UNIT-03

Q.1	Write Euler's formulae for Fourier Series.
Q.2	State Parseval's Theorem.
Q.3	Write Dirichlet's conditions.
Q.4	What is Fourier's Series.

UNIT-04

Q.1	if $f = y^x$, Find $\frac{\partial^2 f}{\partial x \partial y}$ at $x = 2, y = 1$
Q.2	Find the directional derivative of the $f(x, y, z) = x^2 + 2y^2 + z$ at the point P(1,1,2) in the direction $\alpha^{\rightarrow} = 3\hat{i} - 4\hat{j}$
Q.3	Find the curl of vector $\vec{V} (x, y, z) = 2x^2\hat{i} + 3z^2\hat{j} + y^3\hat{k}$ at $x = y = z = 1$
Q.4	Velocity vector of a flow field is given as $\vec{V} (x, y, z) = 2xy\hat{i} - 3x^2z\hat{j}$. Find its vorticity vector.



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Q.5	Evaluate $\lim_{(x,y) \rightarrow (0,0)} \frac{2x^3 - y^3}{x^2 + y^2}$ and $\lim_{(x,y) \rightarrow (2,3)} \frac{x^2 + y^3}{2x^2 y}$
Q.6	if $f(x, y, z) = 2x^2 y - y^3 z^2$, then find the $\text{grad} f$ at $(1, -2, 1)$
Q.7	Suppose the force field $\vec{F} = \nabla f$, where $f(x, y, z) = \frac{1}{x^2 + y^2 + z^2}$. Find the work done by \vec{F} in moving the object from $(1, 0, 0)$ to $(0, 0, 2)$



Q.8	if $u = \tan^{-1} \frac{x+y}{x-y}$ prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$
Q.9	Prove that the $\vec{F} = (y^2 \cos x + z^2) \hat{i} + (2y \sin x - 4) \hat{j} + (3xz^2 + 2) \hat{k}$ is a conservative field.
Q.10	Find the first order partial differential coefficient of $x^3 + x^2y^2 + y^4 + 3z^3 - xyz$
Q.11	Define homogeneous functions and write Euler's Theorem for homogeneous function.
Q.12	Write necessary and sufficient condition for Maxima and Minima of function of two variables.
Q.13	Define Lagrange's Multiplier Method.
Q.14	Write the equation of tangent line and normal plane to a given surface.
Q.15	Find the gradient of $f(x, y, z) = x^2y^2 + xy^2 - z^2$ at $(3, 1, 1)$

Integral

Calculus

UNIT- V

Q.1	Write the coordinate of centre of gravity of a solid	RTU - 2019
Q.2	Write the statement of stoke's theorem, Gauss divergence theorem & Green's theorem.	RTU -2023-2024
Q.3	Find $\iint_S \vec{r} \cdot \hat{n} ds$ where S is a closed surface enclosing volume V & $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$	RTU-2018
Q.4	Evaluate (i) $\int_0^1 \int_0^x \int_0^z x^2yz^2 dx dy dz$	
Q.5	Evaluate $\int_0^b \int_0^x xy dx dy$	



Volume & surface of solid of Revolution

UNIT- I

Q.1	Find Volume & Surface of the solid formed by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, about major axis RTU 2016
Q.2	Find the Volume of the solid generated by the revolution of the loop of the curve $(a + x)y^2 = x^2(a - x)$, about x axis.
Q.3	Find the surface & Volume of the solid formed by revolving the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ about x axis.
Q.4	Find the Surface & Volume of the solid generate by revolving the tractrix $x = a \cos t + \frac{a \log(\tan^2 t)}{2}$; $y = a \sin t$ about x axis or about its asymptote RTU-2015
Q.5	Find the Volume & surface generated by revolving the cardioids $r = a(1 + \cos \theta)$ about the initial line.
Q.6	Show that the volume of spherical cap of height h cut off from sphere of radius a is $\pi h^2(a - \frac{h}{3})$
Q.7	Prove that the volume of a frustum of a sphere of height h & the radii r_1 & r_2 is $\frac{\pi}{6} h [3(r_1^2 + r_2^2) + h^2]$
Q.8	The part of the parabola $y^2 = 4ax$, cut off by latus rectum revolves about the tangent at the vertex find the curved surface area of the reel thus generated.
Q.9	Find the volume of the solid formed by revolution of each of the following cycloids (i) $x = a(\theta + \sin \theta)$; $y = a(1 + \cos \theta)$, about x axis (ii) $x = a(\theta - \sin \theta)$; $y = a(1 - \cos \theta)$, about its base.
Q.10	Prove that surface & volume of the solid generated by revolving the loop of the curve $x = t^2$, $y = t - \frac{t^3}{3}$ about x axis are respectively 3π & $\frac{3\pi}{4}$.
Q.11	Show that the volume of the frustum of a cone of height h & with ends radii a & b is $\frac{\pi h}{3} (a^2 + ab + b^2)$
Q.12	Find volume of the solid generated by the revolution of the curve $x = a \cos^3 \theta$, $y = a \sin^3 \theta$ about the x - axis.
Q.13	The region in the first quadrant enclosed by the y -axis and the graphs of $y = \cos x$ and $y = \sin x$ is revolved as out the x - axis to form a solid find its volume.



Q.1	Evaluate $\int_0^\infty \frac{x}{1+x^6} dx$
Q.2	Evaluate $\int_0^\infty \frac{x^5(1-x^6)}{(1+x^6)^2} dx$
Q.3	Evaluate $\int_0^\infty x \bar{e}^x dx$
Q.4	Show that $\int_0^2 x(8-x^3)^{1/3} dx = \frac{16\pi}{9\sqrt{3}}$
Q.5	Evaluate $\int_0^1 x^m (\log_e x)^n dx$, $n > 0$, $m > -1$
Q.6	Evaluate $\int_0^\infty \frac{1}{1+x^4} dx$
Q.7	For $c > 1$, Prove that $\int_0^\infty x^c dx = \frac{c+1}{(\log c)c+1}$
Q.8	Prove that $\Gamma(1 + \frac{1}{n}) = \int_0^\infty e^{-x} x^{n-1} dx$, Hence deduce that $\int_0^\infty e^{-x} dx = \sqrt{\pi}$.
Q.9	$\int_0^\infty \cos x^2 dx$
Q.10	Show that $\frac{1}{2} = \sqrt{\pi}$
Q.11	Show that $\int_0^\infty x^{m-1} \bar{e}^{ax} \cos bx dx = \frac{\cos m\theta}{(a^2+b^2)^{m/2}}$ and $\int_0^\infty x^{m-1} \bar{e}^{ax} \sin bx dx = \frac{\sin m\theta}{(a^2+b^2)^{m/2}}$ where $\theta = \tan^{-1}(\frac{b}{a})$
Q.12	Show that $\int_0^\infty x^{m-1} \cos bx dx = \frac{m}{b^m} \cos(\frac{m\pi}{2})$ and $\int_0^\infty x^{m-1} \sin bx dx = \frac{m}{b^m} \sin(\frac{m\pi}{2})$.
Q.13	Show that $\int_0^\infty \cos(bx^{1/n}) dx = \frac{1}{bn} \cos(\frac{n\pi}{2})$
Q.14	Prove that: $B(m, n) = \frac{\Gamma(m) \Gamma(n)}{\Gamma(m+n)}$
Q.15	Prove that $\Gamma(n) \Gamma(1-n) = \frac{\pi}{\sin n\pi}$, $0 < n < 1$
Q.16	Show that (i) $\Gamma(n+1) = n\Gamma(n)$ (ii) $B(m, n) = B(n, m)$
Q.17	Prove that $\int_{an}^{\infty} \bar{e}^{ax} x^{n-1} dx$



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Q.18

Prove that $\int_0^{\pi/2} \cos^m \theta \sin^n \theta d\theta =$

$$\frac{\Gamma((m+1)/2) \Gamma((n+1)/2)}{2\Gamma((m+n+2)/2)}$$

Q.19

Show that $\underline{B(m,n)} = B(m+1, n) + B(m, n+1)$

Dr. Indu Gupta



Q.20	Prove that $\int_0^{\pi/2} \tan^n x dx = \frac{\pi}{2} \sec^{\frac{n\pi}{2}}$
Q.21	Evaluate $\int_0^{\pi/6} \cos^4 3\theta \sin^2 6\theta d\theta$
Q.22	$\int_0^1 x^4 (1-x^2)^{1/2} dx$
Q.23	$\int_0^{\pi/2} \sin^6 x dx$
Q.24	$\int_0^{\infty} \frac{x}{1+x^6} dx$
Q.25	$\int_0^{\infty} x^8 \frac{(1-x^6)}{(1+x^2)^{2\mu}} dx$
Q.26	$\int_0^{\infty} \sqrt[3]{x} e^{-x} dx$
Q.27	Show that $\int_0^2 x(8-x^3)^{1/3} dx = \frac{16\pi}{9\sqrt{3}}$
Q.28	$\int_0^1 x^m (\log x)^n dx$
Q.29	$\int_0^{\infty} \frac{1}{1+x^4} dx$
Q.30	For C>1 prove that $\int_0^{\infty} cx^c dx = \frac{\Gamma(c+1)}{(\log c)^{n+1}}$
Q.31	Prove that $\Gamma(1 + \frac{1}{n}) = \int_0^{\infty} e^{-x} x^{n-1} dx$



QUESTION

Sequence & Series

Fest the convergence of the following series

Q.1	$1 + \frac{1}{2} + \frac{1.3}{2.4} + \frac{1.3.5}{2.4.6} + \dots$
Q.2	$\frac{\sqrt{\frac{1}{23}} + \sqrt{\frac{2}{33}} + \sqrt{\frac{3}{43}} + \sqrt{\frac{4}{53}} + \dots}{1}$
Q.3	$x + \frac{1}{x-1} + \frac{1}{x+1} + \frac{1}{x-2} + \frac{1}{x+2} + \dots$
Q.4	$xn = \sqrt{n^2 + 1} - \sqrt{n^2 - 1}$
Q.5	$xn = \frac{1 - n^2}{(1 + n)}$
Q.6	$x + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \frac{x^4}{4.5} + \dots$
Q.7	$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \dots$
Q.8	$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{2}{x} + \frac{3}{x^2} + \frac{4}{x^3} + \dots$
Q.9	$\sum \frac{nn^2}{(n+1)^{n^2}}$
Q.10	<p>(i) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$</p> <p>(ii) $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$</p> <p>(iii) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$</p> <p>(iv) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$</p>
Q.11	$x_n = \frac{n^n}{n!}$
Q.12	$2x + 3^2x^2 + 4^3x^3 + \dots$
Q.13	$1 + \frac{x}{7} + \frac{3.6}{7.10} + \frac{x^2}{7.10.13} + \frac{3.6.9}{7.10.13} + \dots$
Q.14	Use taylor's theorem to expand $\sin x$ in powers of $(x - 1)$ and up to second degree terms.



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QUESTION

Q.15	<p>Test the convergence of the series</p> $\sum_{n=1}^{\infty} 1^2 \cdot 5^2 \cdot 9^2 \dots \dots (4n-3)^2$ $\sum_{n=1}^{\infty} 4^2 \cdot 8^2 \cdot 12^2 \dots \dots (4n)^2$	RTU - 2019
Q.16	<p>Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{\sqrt{n} x^n}{\sqrt{n^2+1}}$</p>	RTU _ 2018
Q.17	<p>Test the convergence / Divergence of the series. $\frac{1.2}{3^2 \cdot 4^2} + \frac{3.4}{5^2 \cdot 6^2} + \frac{5.6}{7^2 \cdot 8^2} + \dots$</p>	RTU-2022



QUESTION-
III

Fourier Series

Q.1	<p>Find the Fourier series to represent</p> $f(x) = x \sin x, \quad 0 < x < 2\pi,$
Q.2	<p>Find the Fourier Series for $f(x) = x + x^2$, $-\pi < x < \pi$, Hence Show that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots$</p>
Q.3	<p>Find the Fourier Series for $f(x) = x^2$, $-\pi < x < \pi$, & Hence prove that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$</p> <p>(i) $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$</p> <p>(ii) $\frac{\pi^2}{12} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$</p> <p>(iii) $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots$</p>
Q.4	<p>If $f(x) = \begin{cases} 0, & -\pi \leq x \leq 0 \\ \sin x, & 0 \leq x \leq \pi \end{cases}$</p> <p>Then show that</p> $f(x) = \frac{1}{\pi} + \frac{\sin x}{2} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos 2nx}{4n^2 - 1}$ <p>Hence Prove that</p> $\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{1}{2}$ $\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \dots = \frac{\pi-2}{4}$ <p style="text-align: right;">RTU- 2018</p>
Q.5	<p>Find Half range sine & cosine series to represent $f(x) = x(\pi - x)$ for $0 \leq x \leq \pi$</p> <p>& also find the sum of $\frac{1}{13} - \frac{1}{33} + \frac{1}{53} - \frac{1}{73} + \dots$</p> <p style="text-align: right;">RTU- 2019</p>
Q.6	<p>Find half range cosine series for the function $f(x) = 2x - 1$; for $0 < x < 1$</p>
Q.7	<p>Find the Fourier series to represent $f(x) = x - x^2$, in the interval $-1 < x < 1$.</p>
Q.8	<p>Find half range sine series for the function $f(x) = x$, in the interval $0 < x < 2$.</p>
Q.9	<p>Expand $f(x) = \cos x$ in a Fourier series in the interval $(-\pi, \pi)$</p>
Q.10	<p>Find half range cosine series for the function $f(x) = (x - 1)^2$; $0 < x < 1$</p> <p>Hence show that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$</p>



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QUESTION-
III

Q.11	Find the Fourier series to represent $f(x) = x $; for $-\pi < x < \pi$,
Q.12	Find the F.S for $f(x) = x^2$ in $(-\pi, \pi)$, using parseval's theorem, prove that $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$

RTU- 2019



Q.13	Obtain Half range sine series for $f(x) = e^x, 0 < x < 1$ 2018	RTU-
Q.14	Find the Fourier series expansion of the following periodic function with period 2π . $f(x) = \begin{cases} \pi + x & \text{if } -\pi < x < 0 \\ 0 & \text{if } 0 \leq x < \pi \end{cases}$ RTU- 2022	
Q.15	$0 < x < 1$ If $f(x) = \begin{cases} \pi(2 - x) & 1 < x < 2 \\ 0 & \text{elsewhere} \end{cases}$ Using half range cosine series, show that $\frac{\pi^4}{96} = 1 + \frac{1}{3^4} + \frac{1}{5^4} + \dots$	



UNIT- IV.

Question on Partial Differentiation

Q.1	If $u = \log(y \sin x + x \sin y)$; then prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$
Q.2	If $u = \frac{x^2 \tan^{-1}(y)}{x} - \frac{y^2 \tan^{-1}(x)}{y}$ then show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}$
Q.3	If $u = e^{xyz}$, then show that $\frac{\partial^2 u}{\partial z \partial y \partial x} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$
Q.4	If $\frac{x^2}{a^2+a} + \frac{y^2}{b^2+b} + \frac{z^2}{c^2+c} = 1$; then show that $\frac{(\partial u)^2}{\partial x} + \frac{(\partial u)^2}{\partial y} + \frac{(\partial u)^2}{\partial z} = 2 \left(x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} \right)$
Q.5	If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then show that — — — (i) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$ (ii) $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = \frac{-9}{(x+y+z)^2}$ (iii) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{-3}{(x+y+z)^2}$
Q.6	If $x^x y^y z^z = c$ then show that $\left(\frac{\partial z}{\partial x} \right)_{\partial x \partial y, x=y=z} = -[x(1 + \log x)]^{-1}$
Q.7	If $\frac{\partial \theta}{\partial r}(r, t) = t^n e^{-r^2} / 4t$, then find the value of n for which $\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial \theta}{\partial r}) = \frac{\partial \theta}{\partial t}$
Q.8	If $u = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$ — Then show that — — — (i) $\frac{x}{\partial^2 u} \frac{\partial u}{\partial x} + \frac{y}{\partial^2 u} \frac{\partial u}{\partial y} + \frac{z}{\partial^2 u} \frac{\partial u}{\partial z} = -u$ (ii) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$
Q.9	Verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$; where u is given by (i) $u = \tan^{-1}(y)$ (ii) $u = \log\left(\frac{x^2 + y^2}{xy}\right)$



Page No. _____
Date _____

Questions on Euler's theorem

Q.1	Verify Euler's theorem for the function. (i) $f(x, y) = \frac{(x+y)}{\sqrt{x+y}}$ (ii) $f(x, y) = \frac{x^{\frac{1}{4}}+y^{\frac{1}{4}}}{x^{\frac{1}{5}}+y^{\frac{1}{5}}}$
Q.2	If $u = \cos^{-1} \left\{ \frac{x+y}{\sqrt{x+y}} \right\}$, then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \frac{1}{2} \cot u = 0$
Q.3	If $u = \sin^{-1} \frac{x}{y} + \tan^{-1} \frac{y}{x}$, then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$
Q.4	If $u = \sin^{-1} \left(\frac{x^{\frac{1}{4}}+y^{\frac{1}{4}}}{\sqrt{x^{\frac{1}{5}}+y^{\frac{1}{5}}}} \right)$, then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{20} \tan u$.
Q.5	If $u = \tan^{-1} \left(\frac{x+y}{\sqrt{x+y}} \right)$, then prove that $x^2 \frac{\partial u}{\partial x} + 2xy \frac{\partial u}{\partial y} + y^2 \frac{\partial u}{\partial y} = \sin^2 2u (1 - 4 \sin^2 u)$
Q.6	If $u = \log \left(\frac{x+y}{\sqrt{x+y}} \right)$, then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3$.
Q.7	If $u = \sin^{-1} \left(\frac{x}{x+y} \right)$, then prove that (i) $x u_x + y u_y = \tan u$ (ii) $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = \tan^3 u$
Q.8	If $u = x \sin^{-1} \left(\frac{y}{x} \right)$, then prove that $x^2 \frac{\partial u}{\partial x} + 2xy \frac{\partial u}{\partial y} + y^2 \frac{\partial u}{\partial y} = 0$
Q.9	If $z = x^n f \left(\frac{y}{x} \right) + y^{-n} f \left(\frac{y}{x} \right)$, then prove that $x^2 \frac{\partial z}{\partial x^2} + 2xy \frac{\partial z}{\partial x \partial y} + y^2 \frac{\partial z}{\partial y^2} + x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = n^2 z$



Questions on Change of Variable (Composite Function) & Total Derivative

Q.1	If $u = x^2 - y^2 + \sin y z$ & $y = e^x$, $z = \log x$ then find value of $\frac{du}{dx}$
Q.2	If $z = x^2 y$ & $x^2 + xy + y^2 = 1$, then find $\frac{dz}{dx}$
Q.3	If $(\cos x)^y = (\sin y)^x$ then prove that $\frac{dy}{dx} = y \tan x + \log(\sin y) \frac{\log(\cos x) - x \cot y}{2}$
Q.4	If $x^3 + y^3 = 3axy$ then find the value of $\frac{dy}{dx}$
Q.5	Find $\frac{dy}{dx}$ when $y^x = \sin x$
Q.6	If $x^y + y^x = a^b$, then find $\frac{dy}{dx}$
Q.7	If $ax^2 + 2hxy + by^2 = 1$, then find $\frac{dy}{dx}$
Q.8	If $u = f(y - z, z - x, x - y)$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$
Q.9	If $u = f(x, y, z)$, Prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$
Q.10	If $z = f(u, v)$, where $u = x^2 - 2xy - y^2$, $v = y$ then prove that $(x + y) \frac{\partial z}{\partial x} + (x - y) \frac{\partial z}{\partial y} = (x - y) \frac{\partial z}{\partial v}$
Q.11	If $z = f(x, y)$, where $x = e^u \cos v$, $y = e^u \sin v$ then show that $y \frac{\partial z}{\partial u} + x \frac{\partial z}{\partial v} = e^{2u} \frac{\partial z}{\partial v}$
Q.12	If $x = \xi \cos \alpha - \eta \sin \alpha$; $y = \xi \sin \alpha + \eta \cos \alpha$, then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \frac{\partial u}{\partial \xi} + \frac{\partial u}{\partial \eta}$
Q.13	If $x = r \cos \theta$, $y = r \sin \theta$, then show that (i) $\frac{\partial u}{\partial x} = \cos \theta \frac{\partial u}{\partial r} - \frac{\sin \theta}{r} \frac{\partial u}{\partial \theta}$ (ii) $\frac{\partial u}{\partial y} = \sin \theta \frac{\partial u}{\partial r} + \frac{\cos \theta}{r} \frac{\partial u}{\partial \theta}$ (iii) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{1}{r} \frac{\partial u}{\partial r}$
Q.14	If $z = f(u, v)$ where $u = lx + my$ & $v = ly - mx$ then prove that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (l^2 + m^2) \left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} \right)$
Q.15	If $f(x, y) = 0$ & $\phi(y, z) = 0$, then show that $\frac{\partial f}{\partial v} \frac{\partial \phi}{\partial z} \frac{dz}{dx} = \frac{\partial f}{\partial x} \frac{\partial \phi}{\partial v}$
Q.16	If $f(p, v, t) = 0$ prove that $\frac{\partial p}{\partial v} \frac{\partial v}{\partial t} \frac{\partial t}{\partial p} = -1$
Q.17	If $f(x, y, z) = 0$ & $\phi(x, y, z) = 0$, then prove that $\frac{\partial \phi}{\partial f} \frac{\partial f}{\partial x} \frac{\partial x}{\partial v} \frac{\partial v}{\partial y} \frac{\partial y}{\partial f} \frac{\partial f}{\partial \phi} \frac{\partial \phi}{\partial z} \frac{\partial z}{\partial v}$



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QUESTION

$$(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}) dx = (\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y})$$

Q.18

If V is a function of u & v where $u = x - y$ & $v = xy$, prove that $x^2 \frac{\partial^2 V}{\partial x^2} + y^2 \frac{\partial^2 V}{\partial y^2} = (x+y) \left(\frac{\partial^2 V}{\partial u^2} + xy \frac{\partial^2 V}{\partial v^2} \right)$

$$(x+y) \left(\frac{\partial^2 V}{\partial u^2} + xy \frac{\partial^2 V}{\partial v^2} \right)$$



Q.19

If $u = f(r)$, where $r^2 = x^2 + y^2$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \frac{z}{r} (r^2 f''(r) + 1 f'(r))$ -

UNIT- IV.

Limit & continuity & Tangent plane & Normal line & Directional derivative, Gradient ϕ , divergence & Curl

<p>Q.1</p>	<p>Evaluate the following limits:</p> <p>(a) $\lim_{\substack{x \rightarrow 2 \\ y \rightarrow 3}} \frac{x^2 + y^3}{2x^2y}$</p> <p>(b) $\lim_{\substack{x \rightarrow 1 \\ y \rightarrow 2}} \frac{x+y^2}{3x+y^2}$</p> <p>(c) $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} (x^2 + y)^{\frac{2}{5}}$</p> <p>(d) $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{xy^2}{x^4+y^2}$</p>
<p>Q.2</p>	<p>$\lim_{x \rightarrow y} \frac{x^3 - y^3}{x^2 - y^2}$; when $x \neq 0, y \neq 0$ If $f(x, y) = \begin{cases} x^2 + y^2 & \text{if } x \neq 0, y \neq 0 \\ 0 & \text{if } x = 0, y = 0 \end{cases}$ Check the continuity of $f(x, y)$ at the origin.</p>
<p>Q.3</p>	<p>Discuss the continuity of the function</p> $f(x, y) = \begin{cases} \frac{x}{\sqrt{x^2 + y^2}} & ; (x, y) \neq (0, 0) \\ 2 & ; (x, y) = (0, 0) \end{cases}$
<p>Q.4</p>	<p>Find the tangent plane & normal line to the surface $f(x, y, z) = x^2 + y^2 + z - 9 = 0$ at $P(1, 2, 4)$</p>
<p>Q.5</p>	<p>Find the equation of tangent plane & Normal line to the elliptic paraboloid $z = 2x^2 + y^2$ at the point $P(1, 1, 3)$</p>



UNIT- IV.

Maxima & Minima

Q.1	Examine the extreme value of (i) $f(x, y) = x^2 + xy + y^2 + 1 + 1$ (ii) $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ (iii) $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$ (iv) $f(x, y) = x^2 + y^2 + 2 + 2$ (v) $f(x, y) = x^3y^2(1 - x - y)$
Q.2	Find the maximum & minimum value of $f(x, y) = \sin x \sin y \sin(x + y)$, $0 \leq x, y \leq \pi$
Q.3	Show that the minimum value of the following function is $3a^2$; if $u = xy + a^3 \frac{(1+x)}{x} \frac{(1+y)}{y}$



UNIT- IV.

Maxima & Minima (Lagrange's Multiplier)

Q.1	Divide a into three parts such that their product is a maximum.
Q.2	Find the dimensions of rectangular box, open at the top of maximum capacity whose surface is 108 sq. cm.
Q.3	Show that volume of the greatest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ is $\frac{8abc}{3\sqrt{3}}$
Q.4	Find the minimum value of $x^2 + y^2 + z^2$, when $ax + by + cz = p$
Q.5	Split 24 into three parts such that the continued product of the first, square of the second & cube of the third may be minimum.
Q.6	Prove that the rectangular solid of maximum volume which can be inscribed in a sphere is a cube .
Q.7	The temperature at any point (x, y, z) in space is given by $T = k xyz^2$, where k is a constant. find the highest temperature on the surface of sphere $x^2 + y^2 + z^2 = a^2$
Q.8	A rectangular box, open at the top, is to have a volume of 32 cubic meters. find its dimensions so that the total surface is minimum.
Q.9	Find the shortest & longest distance from the point $(1,2,-1)$ to the sphere $x^2 + y^2 + z^2 = 24$
Q.10	Find the maxima & minima of $u = x^2 + y^2 + z^2$ subject to conditions $ax^2 + by^2 + cz^2 = 1$ & $lx + my + nz = 0$
Q.11	Find the maxima & minima of $f(x, y, z) = x^2 + y^2 + z^2$ where $ax^2 + by^2 + cz^2 = 1$
Q.12	Find the shortest distance from origin to the surface $xyz^2 = 2$



UNIT- IV.

Directional Derivative

Q.1	Find the directional derivative of the function $f(x, y) = x^2y^3 - 4y$ at $P(2, -1)$ in the direction of the vector $\vec{v} = 2\hat{i} + 5\hat{j}$
Q.2	Find the gradient of $f(x, y) = y \log x$ at $P(1, -3)$ & hence obtain the directional derivative of f in the direction of the vector $\vec{u} = \langle \frac{-4}{5}, \frac{3}{5} \rangle$
Q.3	Find the values of a , b , & c such that $\vec{A} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational vector field. Also find its scalar potential.
Q.4	A fluid motion is given by $\vec{q} = (y + z)\hat{i} + (z + x)\hat{j} + (x + y)\hat{k}$ Is this motion irrotational? if so find the velocity potential .
Q.5	Show that the vector field defined by $\vec{A} = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$ is irrotational. Find the scalar potential ϕ such that $\vec{A} = \vec{g} \text{ rad } \phi$
Q.6	Find the angle between the tangent plane to the surface $x \log z = y^2 - 1$ & $x^2y = 2 - z$ at the point $(1, 1, 1)$
Q.7	Find an unit vector normal to surface $x^2y + 2xz = 4$ at the point $(2, -2, 3)$
Q.8	Find the directional derivative of $\frac{1}{r}$ in the direction of \vec{r}
Q.9	Find the angle between the surfaces $f \equiv xy^2z - 3x - z^2 = 0$ and $\phi \equiv 3x^2 - y^2 + 2z - 1 = 0$ at $p(1, -2, 1)$
Q.10	Find a & b so that the surfaces $f = 4x^2y + z^3 - 4 = 0$ & $\phi = ax^2 - byz - (a + 2)x = 0$ may be orthogonal at $p(1, -1, 2)$
Q.11	Find the directional derivative of $\phi = x^2 - 2y^2 + 4z^2$ at $(1, 1, -1)$ in the direction of vector $2\hat{i} + \hat{j} - \hat{k}$. also find the direction of maximum directional derivative at $(1, 1, -1)$ & its maximum value.
Q.12	Find the values of constants a, b, c so that the directional derivative of $\phi = axy^2 + byz + cz^2x^3$ at $p(1, 2, -1)$ has maximum magnitude 64 in the direction parallel to z axis .
Q.13	Find divergence & curl of vector $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$



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Q.14

Find the value of λ , if $\vec{F} = (2x - 5y)\hat{i} + (x + \lambda y)\hat{j} + (3x - z)\hat{k}$ is solenoidal.



UNIT- IV.

Proof of

Q.1	<p style="text-align: center;">Identities</p> <p>If \vec{a} & \vec{b} are vector function then show</p> <p>that (i) $\nabla \cdot (\vec{a} \times \vec{b}) = \vec{b} \cdot (\nabla \times \vec{a}) - \vec{a} \cdot (\nabla \times \vec{b})$</p> <p>(ii) $\nabla \times (\vec{a} \times \vec{b}) = (\vec{b} \cdot \nabla) \vec{a} - \vec{b} \cdot (\nabla \cdot \vec{a}) - (\vec{a} \cdot \nabla) \vec{b} + \vec{a} \cdot (\nabla \cdot \vec{b})$</p> <p>(iii) $\nabla \cdot (\vec{a} \cdot \vec{b}) = (\vec{b} \cdot \nabla) \vec{a} + \vec{a} \times (\nabla \times \vec{b}) + (\vec{a} \cdot \nabla) \vec{b} + \vec{b} \times (\nabla \times \vec{a})$</p>
Q.2	<p>Prove that $\nabla \times \left(\frac{\vec{r} \times \vec{a}}{r^3} \right) = \frac{\vec{a}}{r^3} - \frac{3}{r^5} (\vec{r} \cdot \vec{a}) \vec{r}$</p>
Q.3	<p>If \vec{v}_1 & \vec{v}_2 are vectors joining the fixed points (x_1, y_1, z_1) & (x_2, y_2, z_2) respectively to a variable point (x, y, z) prove that</p> <p>(i) $\text{Div} (\vec{v}_1 \times \vec{v}_2) = 0$</p> <p>(ii) $\text{grad} (\vec{v}_1 \cdot \vec{v}_2) = \vec{v}_1 + \vec{v}_2$</p> <p>(iii) $\text{curl} (\vec{v}_1 \times \vec{v}_2) = 2(\vec{v}_1 - \vec{v}_2)$</p>
Q.4	<p>If \vec{a} is a constant vector, show that</p> <p>(i) $\nabla \cdot (\vec{a} \cdot \vec{v}) = (\vec{a} \cdot \nabla) \vec{v} + \vec{a} \times (\nabla \times \vec{v})$</p> <p>(ii) $\nabla \times (\vec{a} \times \vec{v}) = \vec{a} \cdot (\nabla \cdot \vec{v}) - (\vec{a} \cdot \nabla) \vec{v}$</p>
Q.5	<p>Prove that $\nabla \cdot \vec{A}^2 = 2(\vec{A} \cdot \nabla) \vec{A} + 2\vec{A} \times (\nabla \times \vec{A})$</p>
Q.6	<p>Prove that</p> <p>(i) $\nabla^2 (1) = 0$</p> <p>(ii) $\nabla \cdot [r \nabla \left(\frac{1}{r^3} \right)] = 3r^{-4}$</p> <p>(iii) $\nabla^2 \left[\nabla \cdot \left(\frac{1}{r^2} \right) \right] = 2r^{-4}$</p> <p>(iv) $\nabla^2 (r^n) = n(n+1)r^{n-2}$</p>
Q.7	<p>Show that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$; where $r = \sqrt{x^2 + y^2 + z^2}$</p>



DOUBLE INTEGRAL

UNIT- V

Q.1	Evaluate $\iint r^3 \sin\theta \cos\theta d\theta dr$ over the region bounded by the cardioid $r = a(1 + \cos\theta)$ above the initial line.
Q.2	Evaluate $\iint xy dx dy$ over the positive quadrant of XY plane subject to $x + y \leq 1$
Q.3	Evaluate the integral $\int_0^\infty \int_0^\infty e^{-(x_2+y_2)} dx dy$ by changing into polar co-ordinates.
Q.4	Evaluate $\int_0^1 \int_{\sqrt{2x-x^2}}^{x^2+y^2} (x^2 + y^2) dx dy$ by changing to polar co-ordinates
Q.5	Evaluate $\iint_{\sqrt{a^2+r^2}}^r$ over the one loop of lemniscate $r^2 = a^2 \cos 2\theta$
Q.6	Find by double integration the area lying inside the cardioids $r = a(1 + \cos\theta)$ and outside the circle $r = a$
Q.7	Find by double integration the area enclosed by $y^2 = 4ax$ and $x^2 = 4ay$
Q.8	Find the volume bounded by coordinate plane and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$
Q.9	Find the volume of the Ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
Q.10	Find the area bounded by the cardioid $r = a(1 + \cos\theta)$ by double integration.
Q.11	Change the order of integration and solve $\int_0^1 \int_0^x e^y dy dx$
Q.12	Evaluate $\int_0^1 \int_{e^x}^1 dx dy$
Q.13	Evaluate $\int_0^\infty \int_x^\infty e^{-y} dx dy$
Q.14	Evaluate $\int_0^1 \int_0^x \int_0^{x+y} (x + y + z) dx dy dz$
Q.15	Find the area, by double integration, bounded by parabola $y^2 = 4ax$ and its latus rectum.
Q.16	Find the area of the region R in the xy-plane enclosed by the circle $x^2 + y^2 = 4$, above the line $y = 1$, and below the line $y = \sqrt{3}x$
Q.17	Evaluate $\int_0^1 \int_0^2 dx dy$
Q.18	Find the Centroid of the region in the first quadrant that is bounded above by the line $y = x$ and below by the parabola $y = x^2$
Q.19	Find the mass, centroid of the tetrahedron bounded by the coordinate planes and the plane $x/a + y/b + z/c = 1$
Q.20	Find the mass of the tetrahedron bounded by the co-ordinates planes and the plane $x/a + y/b + z/c = 1$, the variable density μxyz .
Q.21	Evaluate $\int_1^2 \int_0^z \int_0^y xyz dx dy dz$
Q.22	Evaluate $\int_0^1 \int_0^x \int_0^x dx dy$ by changing into polar co-ordinates.
Q.23	Change the order of integration of the following $\int_0^4 \int_x^{2\sqrt{x}} f(x, y) dx dy$



Line integral, surface integral, volume integral

UNIT- V

Q.1	<p>A vector field is given by $\vec{F} = \sin y \hat{i} + x(1 + \cos y) \hat{j}$. Evaluate $\int \vec{F} \cdot d\vec{r}$ over a circular path given by $x^2 + y^2 = a^2$, $z = 0$</p>
Q.2	<p>Find the work done in moving a particle once round a square C, formed by the lines $y = \pm 1$, $x = \pm 1$ in xy plane if force is given by $\vec{F} = (x^2 + xy + z) \hat{i} + (x^2 + y^2 - z) \hat{j} + xy \hat{k}$</p>
Q.3	<p>Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = (3x^2 + 6y) \hat{i} - 14yz \hat{j} + 20xz^2 \hat{k}$ & C is the path joining $(0,0,0)$ to $(1,1,1)$ according to following path i) $(0,0,0)$ to $(1,0,0)$; $(1,0,0)$ to $(1,1,0)$ then $(1,1,0)$ to $(1,1,1)$ ii) Direct straight line joining $(0, 0, 0)$ to $(1, 1, 1)$ iii) along $x=t$, $y=t^2$, $z=t^3$</p>
Q.4	<p>Show that the $\vec{F} = (2xy + z^3) \hat{i} + x^2 \hat{j} + 3z^2 x \hat{k}$ is a conservative field. Find its scalar potential and hence find work done in moving a particle from $(1, -2, 1)$ to $(3, 1, 4)$.</p>
Q.5	<p>If $\phi = 2xyz^2$; $\vec{A} = xy \hat{i} - zj + x^2 \hat{k}$ and C is the curve $x=t^2$, $y=2t$, $z=t^3$ from $t=0$ to $t=1$. Evaluate the line integral. (a) $\int_C \phi \cdot d\vec{r}$ (b) $\int_C \vec{A} \cdot d\vec{r}$</p>
Q.6	<p>Find the total work done in moving a particle in a force field given by $\vec{F} = 3xy \hat{i} - 5z \hat{j} + 10xk$ along the curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from $t=1$ to $t=2$</p>
Q.7	<p>Find the work done by a force $\vec{F} = (y^2 \cos x + z^3) \hat{i} + (2y \sin x - 4) \hat{j} + (3xz^2 + z) \hat{k}$ in moving a particle from P $(0,1,-1)$ to Q $(\frac{\pi}{2}, -1, 2)$</p>
Q.8	<p>Suppose the force field $\vec{F} = \nabla f$ is the gradient of the function $f(x, y, z) = \frac{-1}{x^2 + y^2 + z^2}$. Find the work done by F in moving an object along a smooth curve joining $(1, 0, 0)$ to $(0, 0, 2)$ that does not pass through the origin.</p>



SURFACE INTEGRAL

UNIT- V

Q.1	Evaluate $\iint_S \vec{A} \cdot \hat{n} ds$ where $\vec{A} = 18z\hat{i} + 12\hat{j} + 3y\hat{k}$ and S is the part of the plane $2x+3y+6z = 12$ which is located in the first octant.
Q.2	If S is the entire surface of cube bounded by $x=0$, $x=a$, $y=0$, $y=a$, $z=0$; $z=a$ and $\vec{A} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$. Then calculate $\iint_S \vec{A} \cdot \hat{n} ds$
Q.3	Evaluate $\iint_S \vec{F} \cdot \hat{n} ds$, where $\vec{F} = 2x^2y\hat{i} - y^2\hat{j} + 4xz^2\hat{k}$ & S is closed surface of the region in the first octant bounded by the cylinder $y^2 + z^2 = 9$ & planes $x=0$, $x=2$, $y=0$ & $z=0$
Q.4	Evaluate $\iint_S \vec{F} \cdot \hat{n} ds$ where $\vec{F} = z\hat{i} + x\hat{j} - 3y^2z\hat{k}$; S is the surface of the cylinder $x^2 + y^2 = 16$ in the first octant between $z=0$ & $z=5$.

INTEGRAL

UNIT- V

Q.1	If $\vec{F} = (2x^2 - 3z)\hat{i} - 2xy\hat{j} - 4x\hat{k}$, then evaluate (i) $\int_V \nabla \cdot \vec{F} dv$ (ii) $\int_V (\nabla \times \vec{F}) dV$, where V is the region bounded by $x = 0$, $y = 0$, $z = 0$ & $2x + 2y + z = 4$.
Q.2	If $\vec{F} = 2z\hat{i} - x\hat{j} + y\hat{k}$, evaluate $\iiint_V \vec{F} dV$ where V is the region bounded by the surfaces $x = 0$, $y = 0$, $x = 2$, $y = 4$, $z = x^2$ & $z = 2$.
Q.3	Evaluate $\iiint_V x^2 dx dy dz$ over the region V enclosed by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = a$.



Stokes Theorem

UNIT- V

Q.1	Verify stokes theorem for $\vec{F} = (x^2 + y - 4)\hat{i} + 3xy\hat{j} + (2xz + z^2)\hat{k}$ over the surface of hemisphere $x^2 + y^2 + z^2 = 16$ above xy plane .
Q.2	Use stokes theorem to evaluate $\int_C \{(x + 2y)dx + (x - z)dy + (y - z)dz\}$ Where C is the boundary of the triangle with vertices (2, 0, 0), (0, 3, 0), and (0, 0, 6), taking anti-clockwise.
Q.3	Verify Stoke's theorem for: $\vec{F} = (y - z + 2)\hat{i} + (yz + 4)\hat{j} - xz\hat{k}$ over the surface of a cube $x = 0, y = 0, x = 2, y = 2, z = 2$ above the xy -plane (cubes bottom is open).
Q.4	Use Stoke's theorem to evaluate: $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = y^2\hat{i} + x^2\hat{j} - (x + z)\hat{k}$ and C is the boundary of the triangle (1, 0, 0), (0, 1, 0), and (0, 0, 1).
Q.5	Apply Stoke's theorem to evaluate: $\int_C (ydx + zdy + xdz)$, where C is the curve of the intersection of $x^2 + y^2 + z^2 = a^2$ and $x + z = a$.
Q.6	Verify Stoke's theorem for the vector field $\vec{F} = (x^2 - y^2)\hat{i} + 2xy\hat{j}$, integrated around the rectangle $z = 0$ and bounded by the lines $x = 0, y = 0, x = a$, and $y = b$.
Q.7	Verify Stoke's theorem for the hemisphere S; $x^2 + y^2 + z^2 = 9, z \geq 0$, its bounding circle C: $x^2 + y^2 = 9, z = 0$, and the field $\vec{F} = y\hat{i} - x\hat{j}$.



GAUSS DIVERGENCE THEOREM

UNIT- V

Q.1	Verify Gauss divergence theorem given that $\vec{F} = 4xzi - y^2 j + yz k$ and S is the surface of the cube bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0$, and $z = 1$.
Q.2	Use divergence theorem to evaluate $\iint_S (x dydz + y dzdx + z dxdy)$ where S is a portion of the plane $x + 2y + 3z = 6$ which lies in the first octant.
Q.3	Verify Gauss divergence theorem for the function $\vec{F} = yi + xj + z^2k$ over the cylindrical region bounded by $x^2 + y^2 = 9, z = 0$ and $z = 2$.
Q.4	Use divergence theorem to evaluate $\iint_S \vec{F} = 4xi - 2y^2j + z^2k$, where S is the surface bounding the region $x^2 + y^2 = 4, z = 0$ and $z = 3$
Q.5	Use Gauss divergence theorem to evaluate $\int_S \vec{F} \cdot \hat{n} ds$ where $\vec{F} = 4xyi + yz j - xz k$ and S is the surface of the cube bounded by the planes $x = 0, x = 2, y = 0, y = 2, z = 0$, and $z = 2$.
Q.6	State Gauss's divergence theorem. Verify Gauss divergence theorem for $\vec{F} = xyi + z^2j + 2yz k$ on the tetrahedron $x = y = z = 0$ and $x + y + z = 1$.



Green's theorem

UNIT- V

Q.1	Apply Green's Theorem to evaluate the line integral of $\oint (xy + y^2) dx + x^2 dy$ where C is the boundary of the closed region bounded by $y=x$ & $y=x^2$
Q.2	Verify Green's theorem in the plane for $\int_C (xdy - ydx)$ Where C is the right angled triangle with Vertices (0,0), (2,0) & (0,1)
Q.3	Verify Green's theorem for $\int_C (x^2 - 2xy) dx + (x^2y + 3)dy$ around the boundary C of the region $y^2=8x$ & $x=2$
Q.4	Evaluate $\int_C (y - \sin x) dx + \cos x dy$ where C is the plane triangle enclosed by the lines $y=0$, $x=\pi/2$ & $y=2x/\pi$
Q.5	Verify Green's theorem for $\int_C (3x^2 - 8y^2) dx + (4y - 6xy)dy$ where C is bounded by parabolas $y=x^2$ & $x=y^2$
Q.6	Verify Green's theorem for $\int_C (3x^2 - 8y^2) dx + (4y - 6xy)dy$ where C is bounded by of the region defined by $x = 0$, $y = 0$ and $x + y = 1$